

Attosecond time delay in photoionization

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PhD: [LTH] -> Post-doc: [SU] -> Guest res.: [CFEL/MPG] -> Researcher [SU]

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AlbaNova

Outline of lecture:

- *Review of attosecond pulse characterization*
 - Simple models based on SFA*
- *How large is the atomic response?*
 - Argon photoionization delay experiment
 - Delays in other noble gas atoms
- *How can we interpret the atomic delays?*
 - Coulomb potential and laser field
 - Many electron effects (“Feynman diagrams”)
 - Autoionization processes
- *Conclusion and Outlook*

* SFA=Strong Field Approximation

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- *Review of attosecond pulse characterization [Basic level]*
 - Simple models based on SFA*
- *How large is the atomic response? [Intermediate level]*
 - Argon photoionization delay experiment
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- *How can we interpret the atomic delays? [State of the art]*
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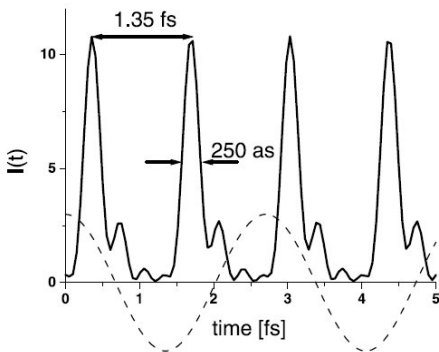
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- **Problems for the PhD-students (Task : i)**

* SFA=Strong Field Approximation

Observation of a Train of Attosecond Pulses from High Harmonic Generation

P. M. Paul,¹ E. S. Toma,² P. Breger,¹ G. Mullot,³ F. Augé,³
Ph. Balcou,³ H. G. Müller,^{2*} P. Agostini¹

In principle, the temporal beating of superposed high harmonics obtained by focusing a femtosecond laser pulse in a gas jet can produce a train of very short intensity spikes, depending on the relative phases of the harmonics. We present a method to measure such phases through two-photon, two-color photoionization. We found that the harmonics are locked in phase and form a train of 250-attosecond pulses in the time domain. Harmonic generation may be a promising source for attosecond time-resolved measurements.



[Paul et al. SCIENCE 1690 292 (2001)]



– “RABIT”, “RABBIT” or “RABBITT”?

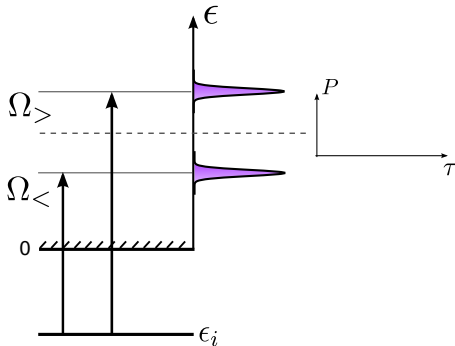
**Now that we have attopulses –
Why is a laser field needed to characterize pulses?**

Group-delay characterization of high-order harmonics

RABBIT method

Linear interaction: $P(\epsilon) \sim |E(\Omega)|^2 |\Psi(\epsilon)|^2$

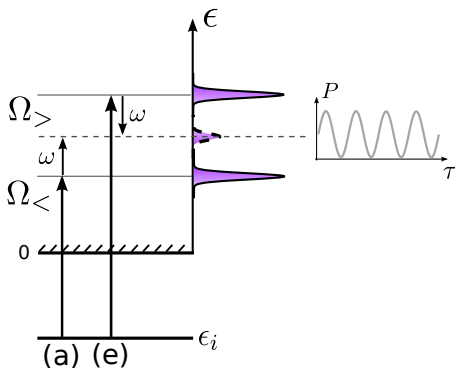
– No phase information about attopulses –



*Photoelectron peaks due to absorption
of one XUV harmonic photon*

$$\Omega_{2q+1} = (2q + 1)\omega$$

Spectral shearing by absorption/emission of laser photon – How the phase of attopulse varies with energy –



Laser-induced sideband signal:

$$P \approx A + B \cos[2\omega(\tau - \tau_{\text{GD}} - \tau_{\text{Atom}})],$$

where $\tau_{\text{GD}} \approx (\phi_{>} - \phi_{<})/2\omega$ is group delay of attopulse

**How can the atomic delay, τ_{Atom} , be determined?
Is it important or negligible?**

Model: atom in multi-color electromagnetic fields

Atomic units: $e = m = \hbar = 4\pi\epsilon_0 = 1$

Hamiltonian for interaction with field *and* ion:

$$H = H_V + V_A$$

Kinetic energy of electron in a *uniform* electromagnetic field:

$$H_V = \frac{1}{2}[\hat{\mathbf{p}} - q\mathbf{A}(t)]^2, \quad q = -1 \text{ au}$$

Atomic potential for hydrogen:

$$V_A(r) = -\frac{1}{r}$$

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Argon potential* within *single-active electron* approximation:

$$V_A(r) = -\frac{1}{r}(1 + 5.4e^{-r} + 11.6e^{-3.682r})$$

* PT: [E S Toma and H G Muller JPB **35**, 3435 (2002)] TDSE: [J Mauritsson et al. PRA **72**, 013401 (2005)]

Amplitude *and* phase of two-photon matrix elements

Table 1. The atomic phases $\Delta\varphi_{\text{atomic}}^f$ and the relative strengths A_f of each two-photon transition responsible for the sideband peaks. The numbers within the parentheses represent the values of the angular and magnetic quantum numbers of the initial 3p state and the final continuum state of the listed energy.

Sideband	$\Delta\varphi_{\text{atomic}}^f$ (rad) / amplitude A_f (arbitrary units)			
	(1,0) \rightarrow (1,0)	(1,0) \rightarrow (3,0)	(1, ± 1) \rightarrow (1, ± 1)	(1, ± 1) \rightarrow (3, ± 1)
$E_0 + 12\hbar\omega$	0.438/6094	0.060/3659	0.125/1914	0.060/2440
$E_0 + 14\hbar\omega$	0.292/5135	0.102/2311	0.125/1281	0.102/1541
$E_0 + 16\hbar\omega$	0.221/3645	0.100/1349	0.108/763	0.100/899
$E_0 + 18\hbar\omega$	0.192/2444	0.090/742	0.090/427	0.090/494

If we know the amplitudes and phases then we can compute τ_{Atom} and deduce the group delay of the attopulses τ_{GD} in experiments.

[Paul et al. SCIENCE 1690 292 (2001)]

Simplest possible model for RABBITT

Atomic units: $e = m = \hbar = 4\pi\epsilon_0 = 1$

*Assume that the photoelectron
is unaffected by the atomic potential:*

Plane wave:

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \exp[i\mathbf{k} \cdot \mathbf{r}]$$

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Momentum eigenstate:

$$\hat{\mathbf{p}}\varphi_{\mathbf{k}} \equiv -i\nabla\varphi_{\mathbf{k}}(\mathbf{r}) = \mathbf{k}\varphi_{\mathbf{k}}$$

Solution to the free particle Schrödinger equation (SE):

$$H_0\varphi_{\mathbf{k}} = \frac{\hat{\mathbf{p}}^2}{2}\varphi_{\mathbf{k}} = \frac{k^2}{2}\varphi_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}}\varphi_{\mathbf{k}}$$

Simplest possible model for RABBITT

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Second-order perturbation theory*:

$$M_{\mathbf{k}}^{(2)} \approx \int d^3k' \frac{\langle \mathbf{k} | O | \mathbf{k}' \rangle \langle \mathbf{k}' | O | g \rangle}{(\epsilon_g + \omega - \epsilon_{\mathbf{k}'})}$$

Perturbation by external field (dipole approximation):

$$\text{Velocity : } O = \mathbf{A}(\omega) \cdot \hat{\mathbf{p}}$$

$$\text{Length : } O = \mathbf{E}(\omega) \cdot \mathbf{r}$$

Vector potential and electric field (uniform in space):

$$\tilde{\mathbf{E}}(t) = -\frac{\partial \tilde{\mathbf{A}}}{\partial t}$$

* In depth discussion: [A Jimenez-Galan, F. Martin and L. Argenti RPA **93**, 023429 (2016)]

Simplest possible model for RABBITT

Atomic units: $e = m = \hbar = 4\pi\epsilon_0 = 1$

(Task : 1) Approximate two photon matrix element:

$$M_{\mathbf{k}}^{(2)} \approx -2A(\Omega)A(\omega) \frac{\epsilon_{\mathbf{k}}}{\omega} \cos^2 \theta_{\mathbf{k}} \langle \mathbf{k} | g \rangle$$

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(Task : 2) Projection of ground state (1s) on plane wave:

$$\langle \mathbf{k} | g \rangle = \frac{2^{3/4}}{\pi} \frac{I_p^{5/4}}{(I_p + \epsilon_k)^2}, \quad I_p = \frac{Z^2}{2}$$

The two-photon matrix goes like $1/\epsilon_k$, $\epsilon_k \gg I_p$
and it is *real* within *plane-wave* approximation:

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and it is *real* within *plane-wave* approximation:

→ The atomic delay is zero!?

**What happens if the laser field
is treated non-perturbatively?**

Electron driven in a field (Volkov state)

Atomic units: $e = m = \hbar = 4\pi\epsilon_0 = 1$

Time-dependent Schrödinger equation (TDSE):

$$i\frac{\partial\psi}{\partial t} = H_V\psi(\mathbf{r}, t)$$

Volkov Hamiltonian (velocity gauge):

$$H_V = \frac{1}{2} [\mathbf{p} + \mathbf{A}(t)]^2$$

Ansatz using plane wave with time-dependent phase:

$$\psi_{\mathbf{k}}^V(\mathbf{r}, t) = \phi_{\mathbf{k}}(\mathbf{r}) \exp[-i\Phi_{\mathbf{k}}(t)]$$

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(Task : 3) Insert into TDSE to obtain the Volkov phase:

$$\Phi_{\mathbf{k}}(t) = \int_{\text{ref.}}^t dt' \frac{1}{2} [\mathbf{k} + \mathbf{A}(t')]^2$$

Photoionization to laser dressed continuum *

Laser-dressed time-dependent perturbation theory *

$$c_{\mathbf{k}}(t) = \frac{1}{i} \int_{-\infty}^t dt' A_X(t') \langle \Psi_{\mathbf{k}}^V | \hat{p}_z | \tilde{g} \rangle$$

where the conjugate Volkov state is

$$\Psi_{\mathbf{k}}^{V*}(\mathbf{r}, t) = \phi_{\mathbf{k}}^*(\mathbf{r}) \exp[i\Phi_{\mathbf{k}}(t)]$$

and the ground state is with binding $I_p > 0$ is

$$\tilde{g}(\mathbf{r}, t) = g(\mathbf{r}) \exp[-i\epsilon_g t] \equiv g(\mathbf{r}) \exp[iI_p t]$$

* [M Kitzler, N Milosevic, A Scrinzi, F Krausz, and T Brabec PRL **88**, 173904 (2002)]

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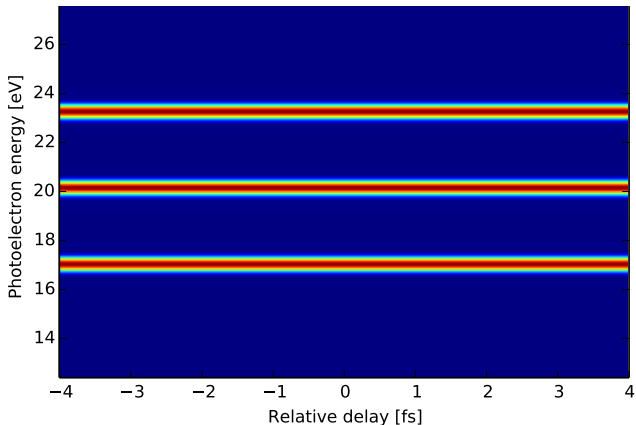
Amplitude for final momentum \mathbf{k} :

$$c_{\mathbf{k}}(t) = \underbrace{\frac{1}{i} \langle \phi_{\mathbf{k}} | \hat{p}_z | g \rangle}_{\text{Independent of } t} \int_{-\infty}^t dt' \underbrace{A_X(t')}_{\text{XUV at } t'} e^{i \int^{t'} dt'' \overbrace{\left\{ \frac{[\mathbf{k} + \mathbf{A}_L(t'')]^2}{2} + I_p \right\}}^{\text{Instantaneous energy}}}$$

* [M Kitzler, N Milosevic, A Scrinzi, F Krausz, and T Brabec PRL **88**, 173904 (2002)]

Photoelectron spectrogram

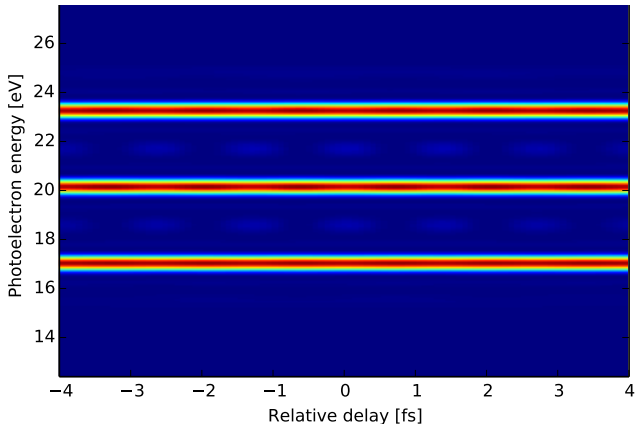
One photon absorption from XUV comb and dressing by laser field (Volkov approx.)



*Redistribution of three harmonic peaks due laser dressing:
Formation of sidebands.*

Photoelectron spectrogram

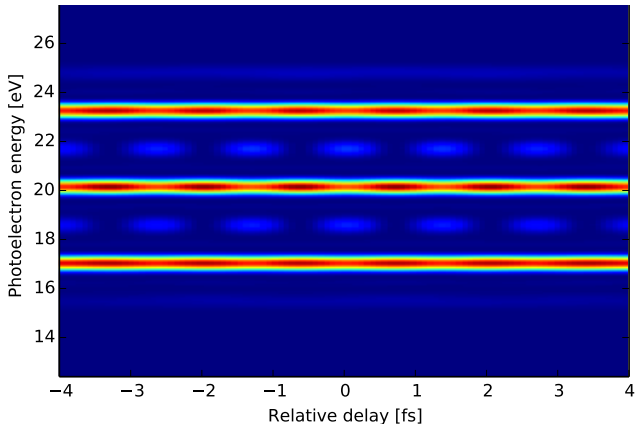
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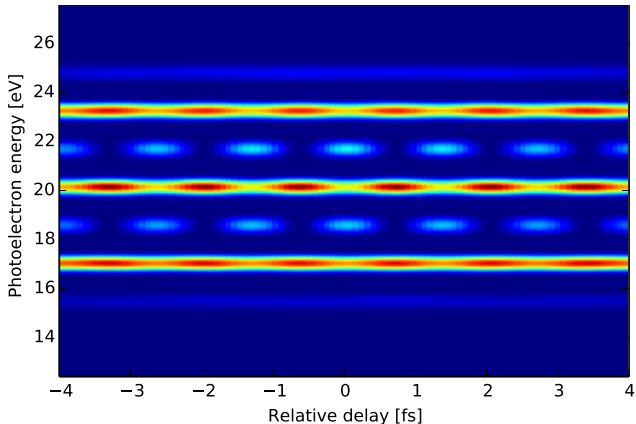
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How does the photon picture arise?

Connection to the photon picture

Amplitude for laser-dressed one-photon ionization:

$$c_{\mathbf{k}}(t) = \frac{1}{i} \langle \varphi_{\mathbf{k}} | \hat{p}_z | g \rangle \int_{-\infty}^t dt' A_X(t') \exp \left[i \int^{t'} dt'' \frac{[\mathbf{k} + \mathbf{A}_L(t'')]^2}{2} + I_p \right]$$

Assume weak laser $[\mathbf{k} + \mathbf{A}_L(t'')]^2 \approx k^2 + 2\mathbf{k} \cdot \mathbf{A}_L(t'')$
and slowly varying *laser* envelope $\Lambda_L(t)$ compared to
laser oscillation ω_L with $A_L(t) = \Lambda_L(t) \sin \omega_L t$

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- Photon energy conservation given by exponential factor.
- Multiphoton transition determined by *real* Bessel function, J_n .

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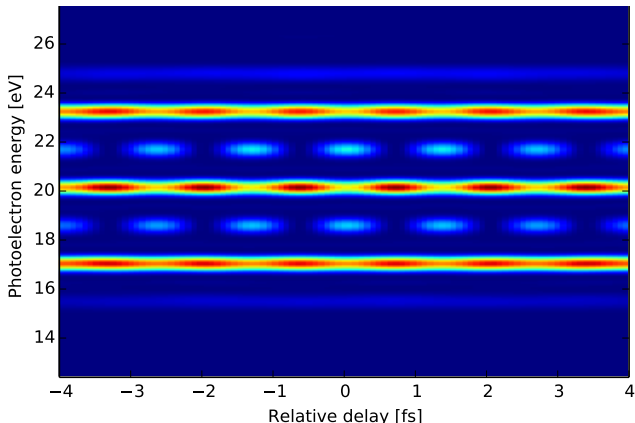
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→ The atomic delay is zero?!?!

Photoelectron spectrogram

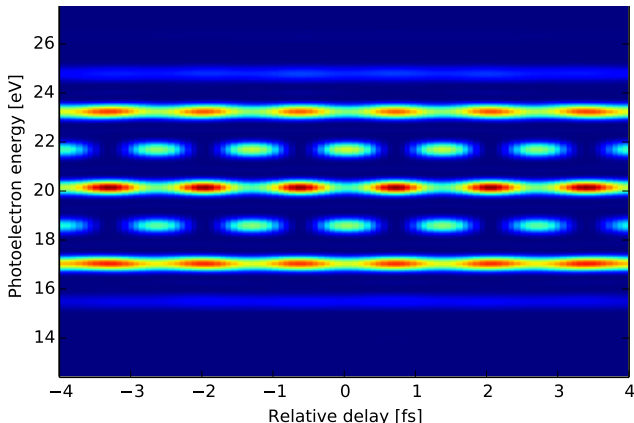
One photon absorption from XUV comb and dressing by laser field (Volkov approx.)



*Redistribution of harmonics due to intense laser dressing.
Multi-photon processes amount to non-sinusoidal beating patterns.*

Photoelectron spectrogram

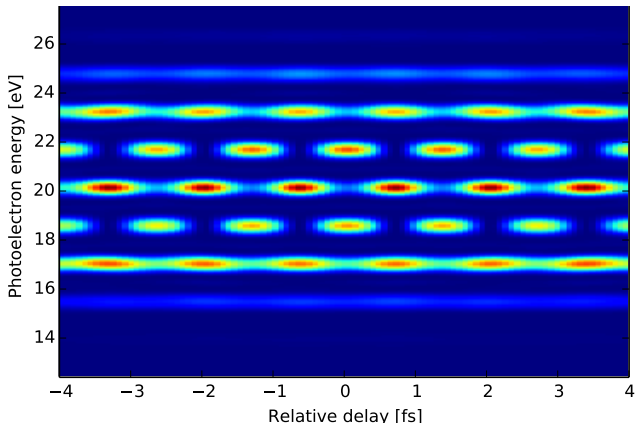
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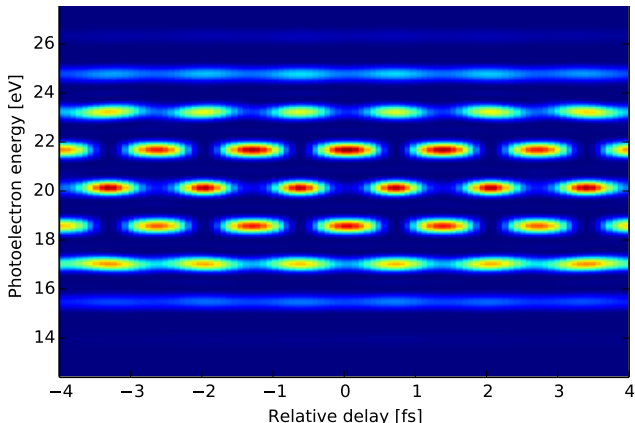
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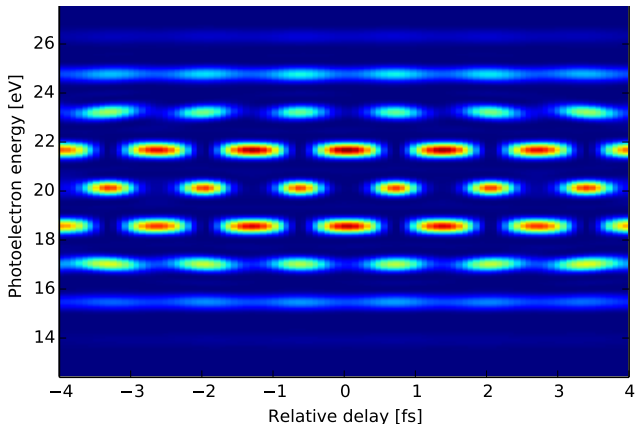
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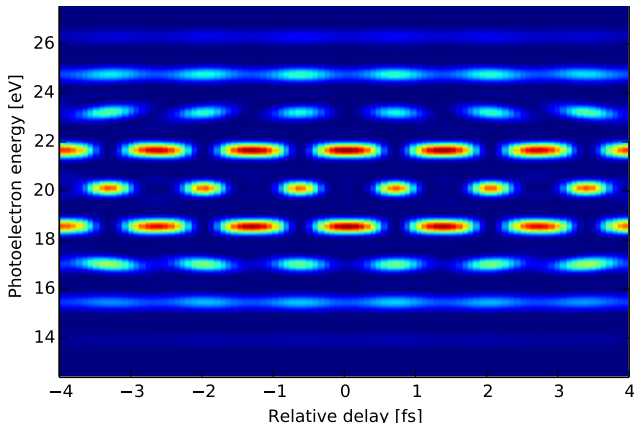
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Group-delay characterization of high-order harmonics

RABBITT method based on higher-order laser photon processes

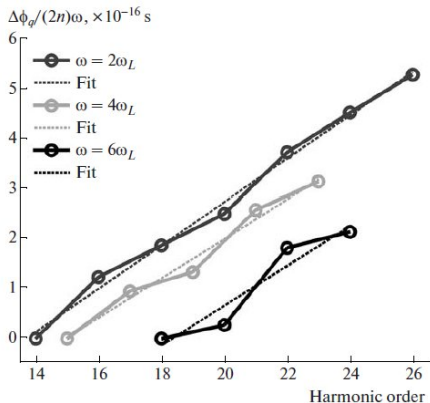


Fig. 8. Comparison of the obtained phase differences for three different frequency components present in the experimental electron signal. The conventional RABBITT includes contribution from sidebands 14 to 26. The $4\omega_R$ -component has been extracted from harmonics 15 to 23 and the $6\omega_R$ -modulation was obtained from sidebands 18 to 24. The curves have been shifted for better comparison.

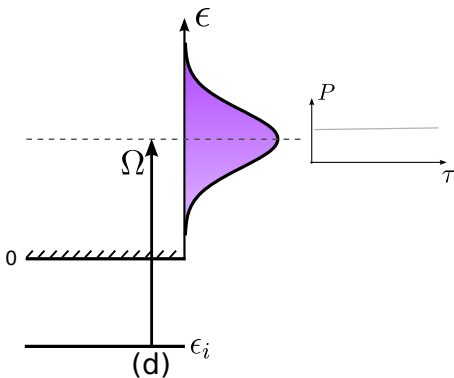
[Swoboda et al. Laser Physics **19** 1591 (2009)]

**What if a single attopulse is used?
(instead of an attosecond pulse train)**

Group-delay characterization of coherent XUV continuum

FROG-CRAB method (=...Complete Reconstruction of Attosecond Burst)

No temporal information by one-photon ionization

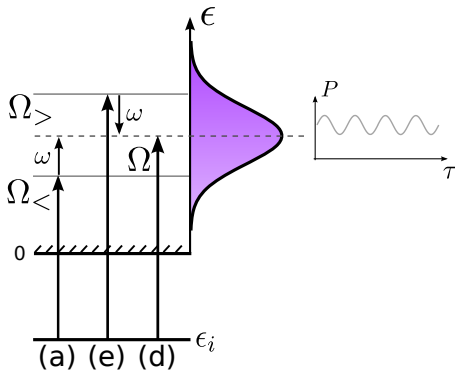


Broad photoelectron peak due to absorption of one XUV harmonic photon Ω

Group-delay characterization of XUV continuum

FROG-CRAB method (=...Complete Reconstruction of Attosecond Burst)

Laser field will induce complex interference

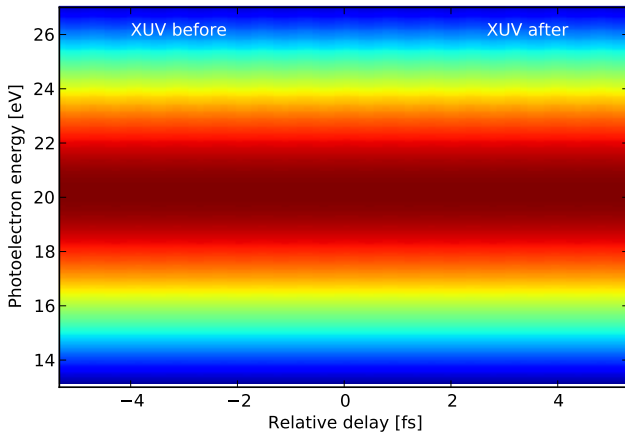


Preferred: *Classical picture "Streaking of photoelectron"*

$$p_f \approx p_0 - A(t_0)$$

Photoelectron spectrogram

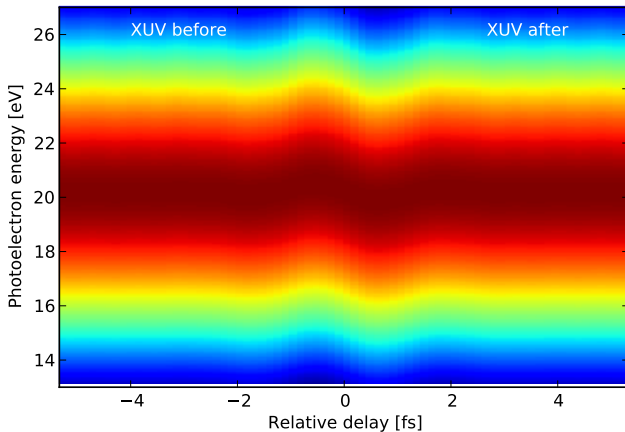
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*Redistribution of photoelectron probability
due to vector potential of laser field*

Photoelectron spectrogram

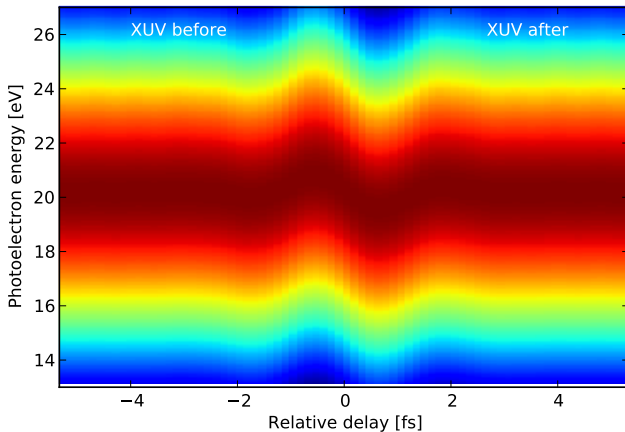
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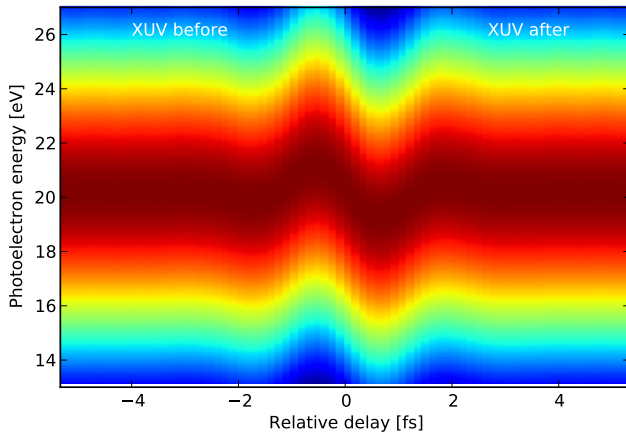
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Connection to the streaking picture

Amplitude for laser-dressed one-photon ionization:

$$c_{\mathbf{k}}(t) = \frac{1}{i} \langle \varphi_{\mathbf{k}} | \hat{p}_z | g \rangle \int_{-\infty}^t dt' A_X(t') \exp \left[i \int^{t'} dt'' \frac{[\mathbf{k} + \mathbf{A}_L(t'')]^2}{2} + I_p \right]$$

Assume *short XUV pulse* given by $A_X(t) = \Lambda_X(t - t_0) \sin \omega_X t$,
then the laser vector potential *appears* static: $t'' \approx t' \approx t_0!$

Connection to the streaking picture

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then the laser vector potential *appears* static: $t'' \approx t' \approx t_0$!

$$\alpha_{\mathbf{k}}(t) \approx \frac{1}{i} \langle \varphi_{\mathbf{k}} | \hat{p}_z | g \rangle \int_{-\infty}^t dt' \frac{1}{2} \Lambda_X(t' - t_0) \exp\{i[\epsilon_k + I_p - \omega_X + \mathbf{k} \cdot \mathbf{A}_L(t_0)]t'\} \quad (\text{Task : 5})$$

- Quasi-static vector potential approximation: $A(t'') \approx A(t_0)$.
- Energy conservation determined by exponential factor.
The shift is given by instantaneous laser vector potential!

Connection between multi-photon and streaking pictures

Identification of streaking mechanism as multi-photon processes:

$$\exp[i\mathbf{k} \cdot \mathbf{A}_L(t_0)t'] \leftrightarrow \sum_{n=-\infty}^{\infty} (-i)^n J_n \left(\frac{\mathbf{k} \cdot \mathbf{A}_L(t')}{\omega_L} \right) \exp[in\omega_L t']$$

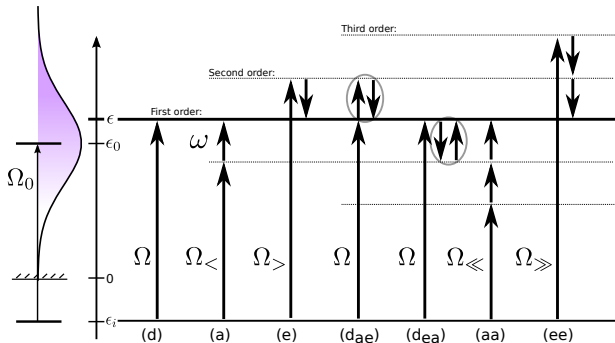
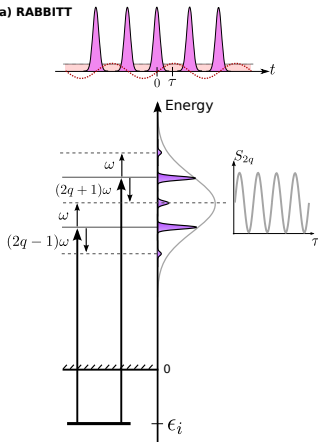


Figure: Multi-photon processes leading to the same final state.

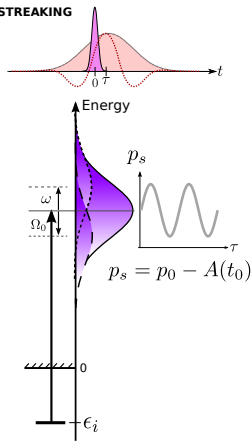
Probing temporal structure of as pulse

Photoelectron is manipulated using an phase-locked laser field

(a) RABBITT



(b) STREAKING



- Spectral shearing interferometry (abs./emi. of laser photon)
- Frequency Resolved Optical Gating (laser sets “phase gate”)

[Paul et al. Science **292**, 1689 (2001)]

[Mairesse and Quéré. PRA, **71** 011401, (2005)]

Can we measure a delay in photoionization?



– It's like trying to measure both the chicken and the egg!

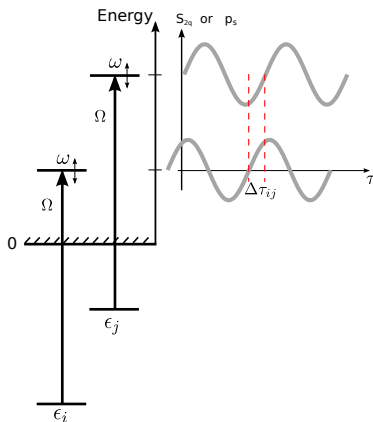
Experimental breakthrough:

- Inter-orbital delay experiments (“between states”)
- Inter-species delay experiments (“between atoms”) using the *same* attosecond pulses.

Inter-orbital photoionization delay experiment

Differential delay between initial orbitals i and j

Idea: Use the same attopulse to ionize from different orbitals!

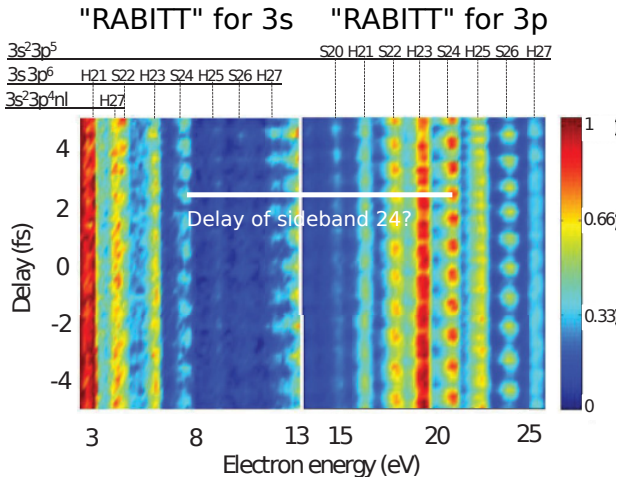


Ne: $2p - 2s$ [Schultze *et al.* Science **328** (2010) 1658]

Ar: $3p - 3s$ [Klünder *et al.* PRL **106** (2011) 143002]

Inter-orbital photoionization delay experiment

(in attoseconds, $1 \text{ as} = 10^{-18} \text{ s}$)

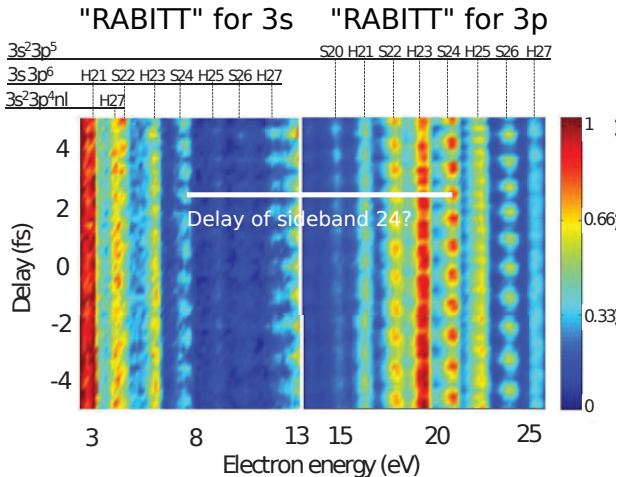


Same sideband order but different ionic states $\sim 100 \text{ as}$ (@37 eV).

[Klünder et al. PRL 106 143002 (2011)] [Guenot et al. PRA **85**,053424 (2012)]

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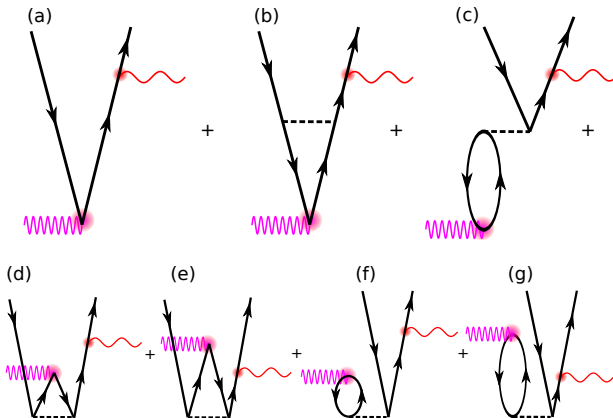


Same sideband order but different ionic states $\sim 100 \text{ as}$ (@37 eV).

Experiment: - "Electrons from 3p are delayed relative 3s."

[Klünder et al. PRL 106 143002 (2011)] [Guenot et al. PRA **85**,053424 (2012)]

Calculation of correlated two-photon matrix elements: (RPAE=Random Phase Approximation with Exchange)



- “Feynman diagrams”: \uparrow =electron and \downarrow =hole
- Absorption of XUV photon with RPAE correlation effects.
- Stimulated electron continuum transition by IR field.

Evaluation of IR-driven continuum transition

The perturbed wavefunction (PWF) is an outgoing wave

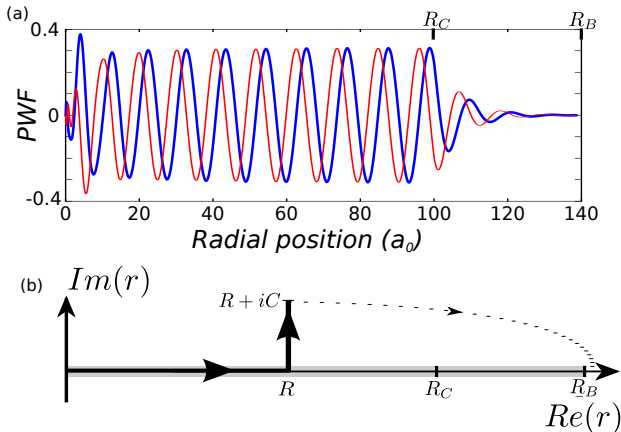
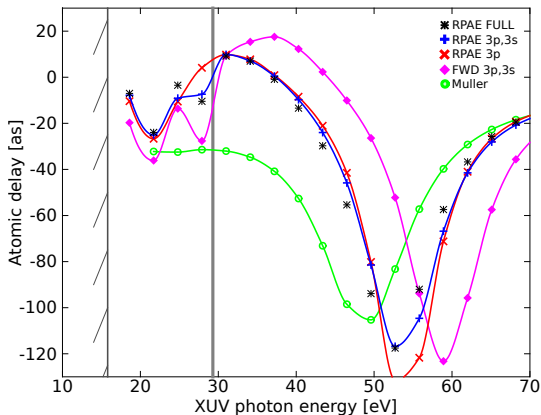


Figure: A perturbed wavefunction (PWF) is setup on **B-splines** (kord=7) with **exterior complex scaled** knot sequence (nknot=250). The PWF is **matched to Coulomb functions** before the scaled region ($x < 100$). The remaining analytical integral is evaluated along the imaginary axis.

Study of correlation effects in $\text{Ar}3p^{-1}$

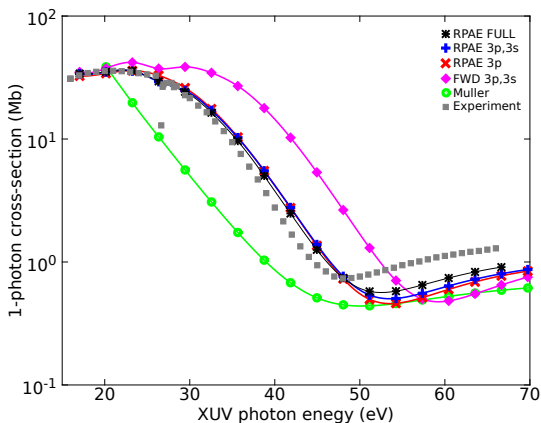
Experimental binding energies (not HF values):



- At 37 eV the atomic delay is small (~ 0 as).
- The atomic delay exhibits a negative peak of ~ -120 as.
- Electron correlation effects amount to ~ 40 as (Muller).

[J M Dahlström and E Lindroth JPB 47 124012 (2014)]

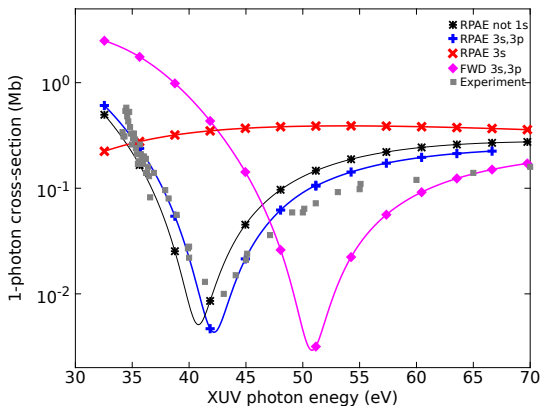
One-photon ionization cross-section for argon [$3p^{-1}$]



- Cooper minimum because dipole matrix element vanishes.*
- Intra-orbital correlation is enough for 3p (6 e^- in 3p orbital).
- Ground-state correlation is important (beyond TDCIS).

*[J W Cooper Phys. Rev. **128** 681 (1962)] Fig: [J M Dahlström and E Lindroth JPB **47** 124012 (2014)]

One-photon ionization cross-section for argon [$3s^{-1}$]

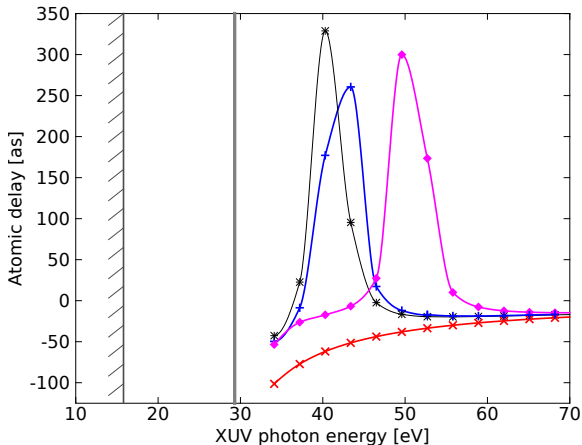


- Cooper minimum in 3s due to correlation with 3p.**
- Intra-orbital correlation between 3s and 3p is required.
- Ground-state correlation is important (beyond TDCIS).

** [M Ya Amusia et al PHYS. LETT. **40A** 361 (1971)] [J M Dahlström and E Lindroth JPB **47** 124012 (2014)]

Study of correlation effects in $\text{Ar}3s^{-1}$

Experimental binding energies (not HF values):



- Large *positive* delay peak (~ 300 as) close to 40 eV^* .
- Electron correlation effects amount to ~ 400 as.
- Here, at 37 eV , the delay is close to zero (~ -15 as).

Comparison between theory and the argon experiment

Table of results for argon delays:

Experiment:	$\tau_{3p} - \tau_{3s} \approx 100$ as	(at 37 eV)
Theory (HF-CC):	$\tau_{3p} - \tau_{3s} \approx 75$ as	(2011)
Theory (RPAE-CC w/ HF):	$\tau_{3p} - \tau_{3s} \approx 78$ as	(2012)

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Other ideas?

- Shake-up processes: $3s^{-1} \rightarrow 3p^{-2}n\ell$.
- The $3s^{-1}$ is only 69% a single hole state.*
- Laser-stimulated hole transitions.**
- Final state correlation (after absorption of IR).
- Or something entirely different? ... More data would be great!

*[T Carette et al. PRA **87**, 023420 (2013)] and **[J A You et al. PRA **93**, 033413 (2016)]

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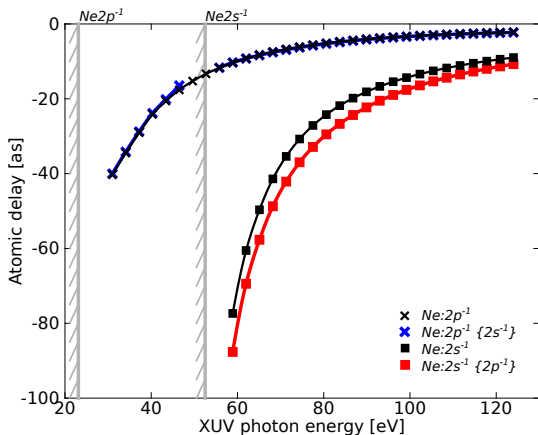
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Theory (RPAE-CC-HH w/ HF):	$\tau_{3p} - \tau_{3s} \approx 100$ as	(Preliminary!)

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“Atomic delays” from 2p and 2s states in Ne



- **Small delay** in 2s due to **inter-orbital** correlation with 2p.
- Delay at ~ 105 eV: $\Delta\tau_{p-s} = 12.4$ as (Exp* ≈ 21 as)
- Delay at 71.3 eV is ~ 36 as (SB:46).

[Dahlström et al. Phys. Rev. A 86, 061402 (2012)], *[Schultze et al. Science 328, 1658 (2010)]

Inter-species photoionization delay experiment

(in attoseconds, $1 \text{ as} = 10^{-18} \text{ s}$)

Sideband	20	22	24
$\tau(\text{Ar}) - \tau(\text{Ne})$	68 ± 15	70 ± 12	52 ± 25
Theory	60	51	40
$\tau(\text{Ar}) - \tau(\text{He})$	82 ± 15	83 ± 22	71 ± 21
Theory	72	59	45
$\tau(\text{Ne}) - \tau(\text{He})$	23 ± 4	12 ± 4	10 ± 8
Theory	12	8	4

- The delay is relative to the same sideband order.
- Ar has a larger delay than both Ne and He.

[Guenot et al. J.Phys.B **47** (2014) 245602]:

Experiment by L'Huillier group at Lund University. Theory by Dahlström and Lindroth at Stockholm University.

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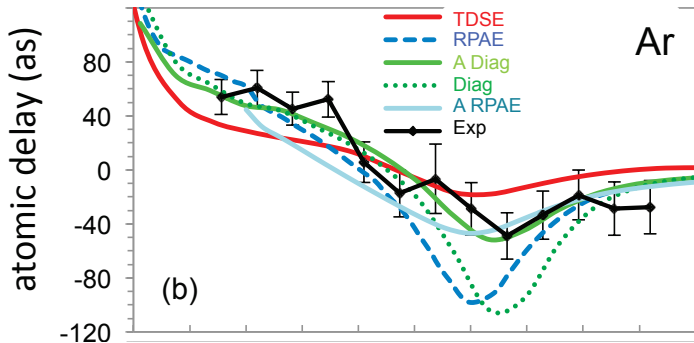
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- Ar has a larger delay than both Ne and He.
- Our calculations show systematically too small delays.

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Experiment by L'Huillier group at Lund University. Theory by Dahlström and Lindroth at Stockholm University.

Relative atomic delay measurements in Ne, Ar and Kr



- The angle-resolved and angle-integrated delays may differ.

Experiment at Ohio State Uni. [C Palatchi et al J. Phys. B: At. Mol. Opt. Phys. 47 (2014) 245003]

- OK, “atomic delays” have been measured experimentally.
Why is it so fascinating — what does it mean?

Probing Single-Photon Ionization on the Attosecond Time Scale

$$\tau_A = \tau_W + \tau_{CC}$$

‘‘The determination of photoemission time delays requires taking into account the measurement process, involving the interaction with a probing infrared field. This contribution can be estimated using a universal formula and is found to account for a substantial fraction of the measured delay.’’

[K. Klünder et al. PRL **106**, 143002 (5 April 2011)]

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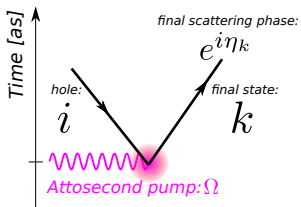
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Time-resolved photoemission by attosecond streaking: extraction of time information

‘‘We show that attosecond streaking ... contain ... Eisenbud-Wigner-Smith time delay matrix ... if ... the streaking infrared (IR) field ... is properly accounted for ...’’ [S Nagele et al. JPB. **44**, 081001 (11 April 2011)]

Photoionization matrix elements

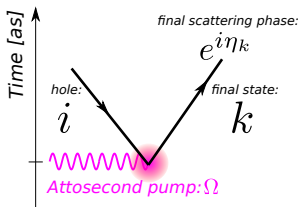


One-photon matrix element:

$$M^1(\vec{k}) = -iE_\Omega \langle \vec{k} | z | i \rangle \\ \sim \exp[i\eta_\ell(k)]$$

[J.M. Dahlström et al Chem.Phys.(2012)]

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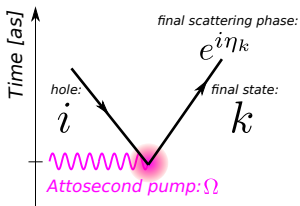
Scattering state expansion in partial wave basis:

$$\phi_{\vec{k}}^{(-)}(\vec{r}) = \sum_{\ell, m} i^\ell e^{-i\eta_\ell} Y_{\ell, m}^*(\hat{k}) Y_{\ell, m}(\hat{r}) R_{k, \ell}(r)$$

Scattering phase, η_ℓ , is specific to the target atom.

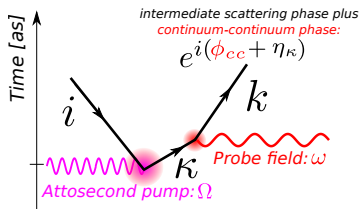
[J.M. Dahlström et al Chem.Phys.(2012)]

Photoionization matrix elements



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Two-photon matrix element:

$$M^2(\vec{k}) = -iE_\Omega E_\omega \\ \times \sum_{\kappa'} \frac{\langle \vec{k} | z | \kappa' \rangle \langle \kappa' | z | i \rangle}{\epsilon_i + \Omega - \epsilon_{\kappa'}} \\ \sim \exp[i\phi_{cc}(k, \kappa) + i\eta_\ell(\kappa)]$$

[J.M. Dahlström et al Chem.Phys.(2012)]

Continuum–continuum phases

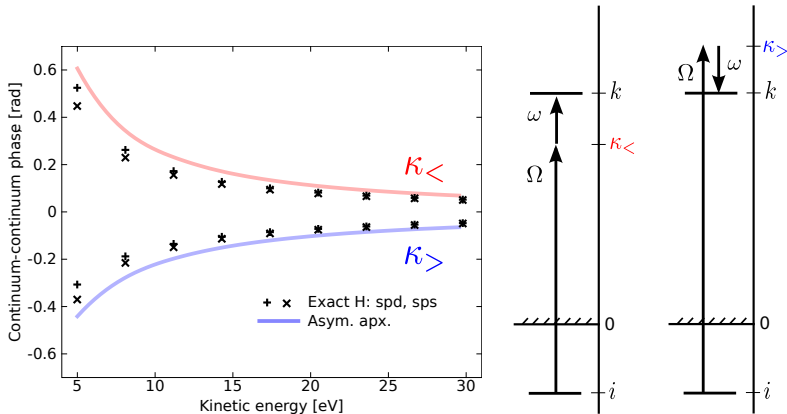


Figure: Exact vs. asymptotic values of $\phi_{cc}(k, \kappa)$.

[K. Klünder *et al.* PRL. (2011)]

Collaboration with A. Maquet and R. Taïeb at UPMC through COST.

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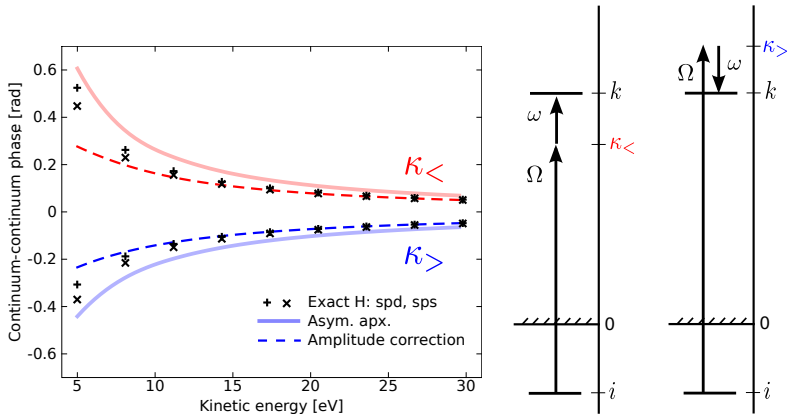


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Phase of 2 photon ATI amplitude

(ATI=Above Threshold Ionization process)

Explicit phase of ATI transition: $i \rightarrow \vec{\kappa} \rightarrow \vec{k}$:

$$\begin{aligned} \arg[M^2(\vec{k})] \approx & \pi + \arg[Y_{L,m_i}(\hat{k})] + \phi_{\Omega} + \phi_{\omega} \\ & - \frac{\pi\ell}{2} + \eta_{\ell}(\kappa) + \phi_{cc}(k, \kappa), \end{aligned}$$

with XUV: Ω first, then continuum–continuum IR: ω .

(One intermediate angular momenta: ℓ .)

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-Now we apply this “ansatz” to experimental schemes!

Applications of ATI matrix elements: "RABBITT"

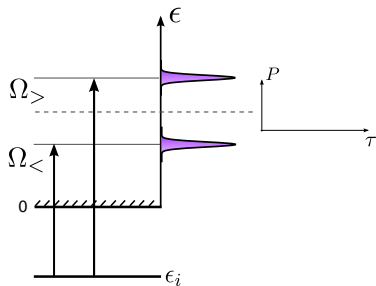
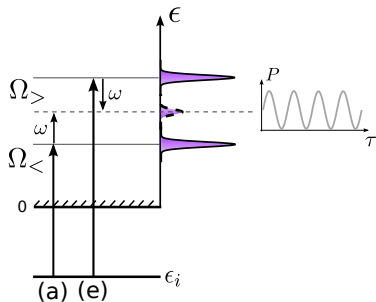


Figure: Ionization by APT.

Applications of ATI matrix elements: “RABBITT”



Probability of emission along \hat{z} :

$$P(\vec{k}) \approx |M_a + M_e|^2$$
$$= |M_e|^2 + |M_a|^2 + 2\Re\{M_e M_a^*\}$$

Figure: Ionization by APT+IR.

Applications of ATI matrix elements: “RABBITT”

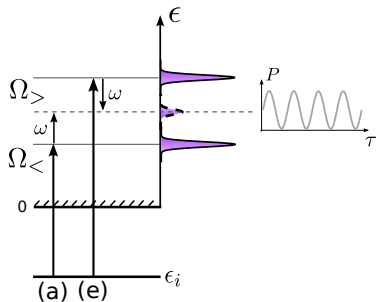


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The phase of the cross-term:

$$\arg\{M_e M_a^*\} \approx -2\omega \times \tau$$

$$+ \phi_{\Omega>} + \eta_{\kappa>,l} + \phi_{cc}(k, \kappa>)$$

$$- \phi_{\Omega<} - \eta_{\kappa<,l} - \phi_{cc}(k, \kappa<)$$

Applications of ATI matrix elements: "RABBITT"

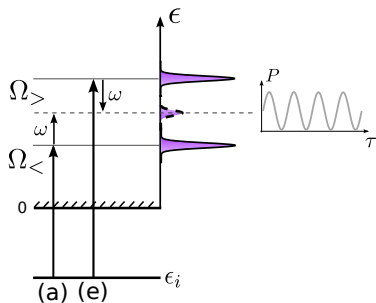


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Applications of ATI matrix elements: "RABBITT"

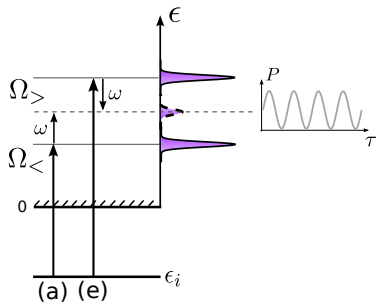


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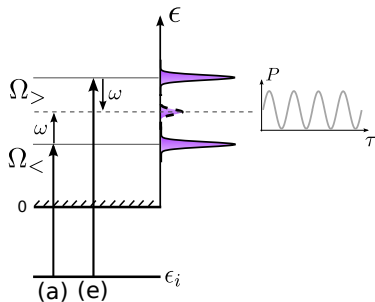
$$= |M_e|^2 + |M_a|^2 + 2\Re\{M_e M_a^*\}$$

Max of modulation!

$$\tau = \frac{\Delta\phi_\Omega}{\Delta\omega} + \frac{\Delta\eta_{\kappa,l}}{\Delta\omega} + \frac{\Delta\phi_{cc}}{\Delta\omega}$$

(Finite-difference derivatives)

Applications of ATI matrix elements: “RABBITT”



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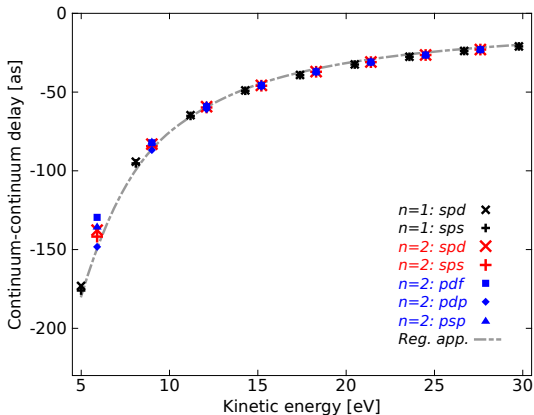
Figure: Ionization by APT+IR.

“RABBITT delays” (relation: max vector potential of probe field vs. arrival of XUV pulse)

$$\tau \approx \tau_\Omega + \tau_{k,l} + \tau_{cc}(k; \omega),$$

Group delay + Wigner delay + Continuum–continuum delay.

Continuum–continuum delays in Hydrogen

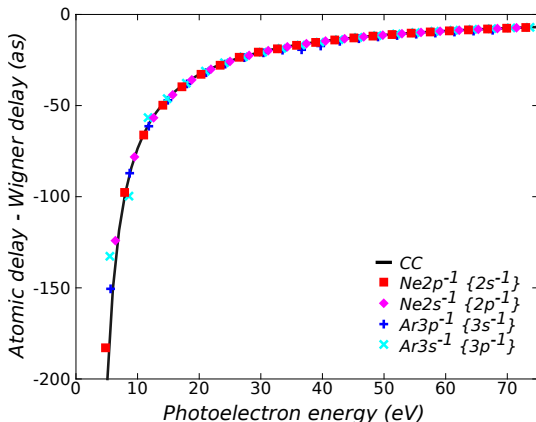


$$\tau_{cc}(k, \omega) \equiv \frac{\Delta\phi_{cc}}{\Delta\omega} \equiv \frac{\phi_{cc}(k, \kappa_{>}) - \phi_{cc}(k, \kappa_{<})}{2\omega}$$

Exact calculations by R. Taïeb (UPMC) for hydrogen using Sturmians.

[J. M. Dahlström and D. Guénot *et al.* Chem. Phys. (2012)]

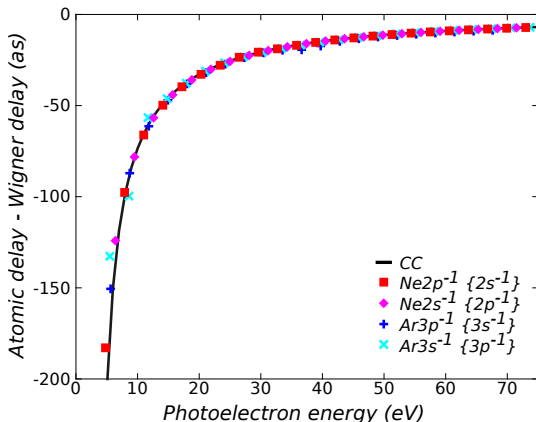
Continuum–continuum delay in Ne and Ar:



“Universal” delay due to continuum–continuum transitions:

$$\tau_{CC} = \tau_A - \tau_W$$

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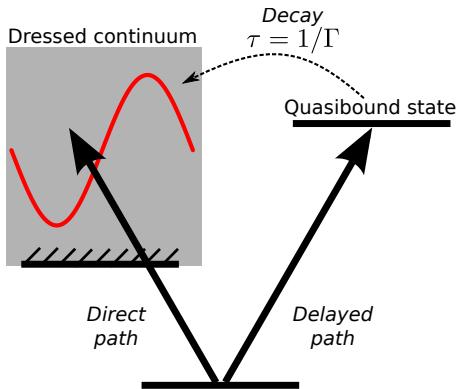
$$\tau_{CC} = \tau_A - \tau_W$$

- Photoelectron detected *along the polarization axis* of fields.
- Photoelectron interacts with laser field (*no stimulated hole*).

What happens if a resonance is embedded in the continuum?

Streaking with a resonance

Direct and autoionizing processes



Asymmetric Fano peak

Photoelectron distribution depends on q -parameter

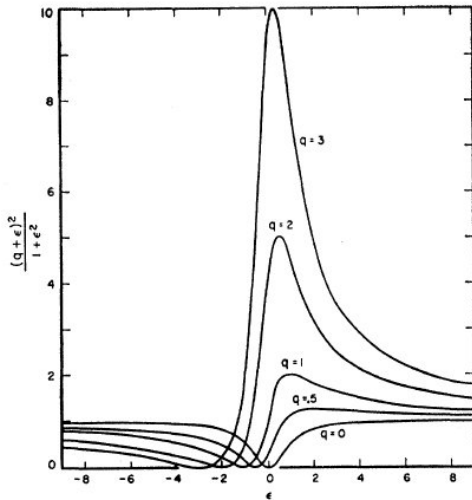


FIG. 1. Natural line shapes for different values of q . (Reverse the scale of abscissas for negative q .)

[U Fano Phys. Rev. 124 1866 (1961)]

Streaking over a resonance

Direct and autoionizing processes

Fano theory transition probability ratio:

$$\frac{|\langle \Psi | T | g \rangle|^2}{|\langle \psi | T | g \rangle|^2} = \frac{(q + \epsilon)^2}{1 + \epsilon^2}$$

where the $\epsilon = (E - E_r)/(\Gamma/2)$ and q describes the resonance.

Corresponding complex amplitude:

$$\langle \Psi | T | g \rangle = \underbrace{\frac{q + \epsilon}{1 - i\epsilon}}_{f_F(E)} \langle \psi | T | g \rangle$$

See: [Z X Zhao and C D Lin PRA **71**, 060702 (2005)]

Streaking over a resonance

Direct and autoionizing processes

Fano theory transition probability ratio:

$$\frac{|\langle \Psi | T | g \rangle|^2}{|\langle \psi | T | g \rangle|^2} = \frac{(q + \epsilon)^2}{1 + \epsilon^2}$$

where the $\epsilon = (E - E_r)/(\Gamma/2)$ and q describes the resonance.

Corresponding complex amplitude:

$$\langle \Psi | T | g \rangle = \underbrace{\frac{q + \epsilon}{1 - i\epsilon}}_{f_F(E)} \langle \psi | T | g \rangle$$

Go to the time domain: (*Task : 6*)

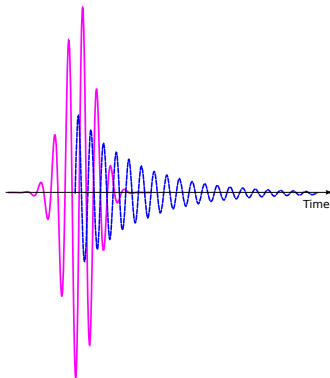
$$F_F(\tau) = \frac{1}{2\pi} \int dE f_F(E) \exp[-iE\tau] = i\delta(\tau) + \frac{\Gamma}{2}(q-i)e^{-iE_r\tau - \Gamma\tau/2} \Theta(\tau)$$

See: [Z X Zhao and C D Lin PRA **71**, 060702 (2005)]

Streaking with a resonance

Direct and autoionizing processes

What would “FROG-CRAB” see?

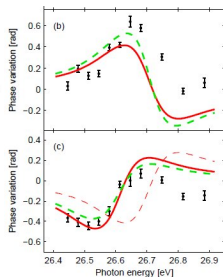
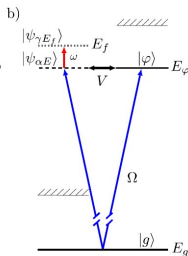
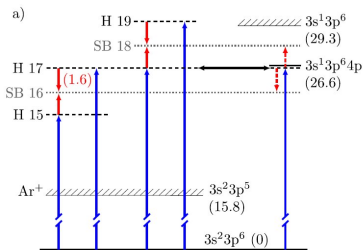


*Effective pulse populating the continuum:
Direct path + Decay (exponential tail)*

See also: [Wickenhauser et al. PRL **94**, 023002 (2005)]
Attosecond transient absorption: Control of Lorentz to Fano: [Ott et al. Science **340**, 716 (2013)]

RABBITT with a resonance

Direct and autoionizing processes



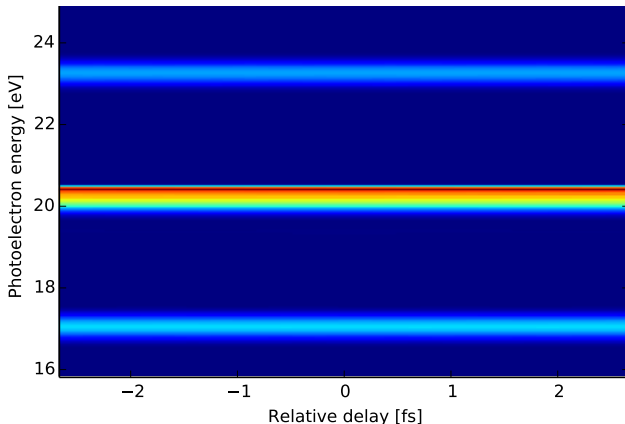
Two-photon matrix element with two continuum and one resonance:

$$M = M^{(1)} \frac{q + \epsilon}{\epsilon + i} + M^{(2)}$$

[Kotur et al. NATURE COMMUNICATIONS — 7:10566 (2015)]

Photoelectron spectrogram

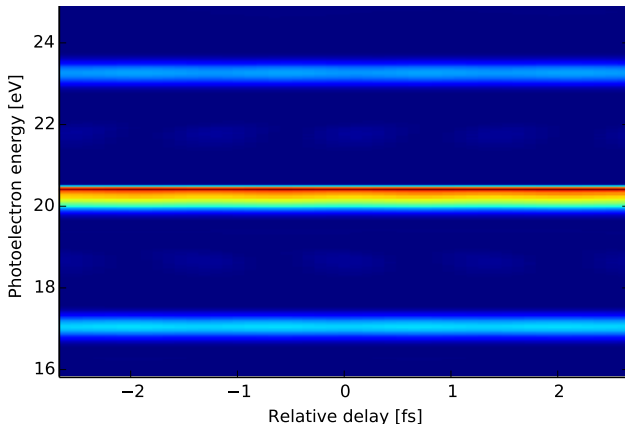
One photon absorption to dressed continuum with autoionization



Redistribution of three harmonic peaks due to laser dressing and autoionizing resonance ($q = -2.75$, $\Delta\epsilon = 0.2\omega_L$ and $\Gamma = 0.1\text{eV}$).

Photoelectron spectrogram

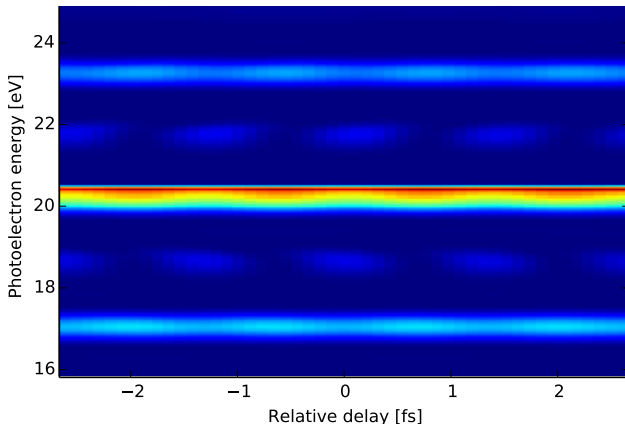
One photon absorption to dressed continuum with autoionization



Redistribution of three harmonic peaks due to laser dressing and autoionizing resonance ($q = -2.75$, $\Delta\epsilon = 0.2\omega_L$ and $\Gamma = 0.1\text{eV}$).

Photoelectron spectrogram

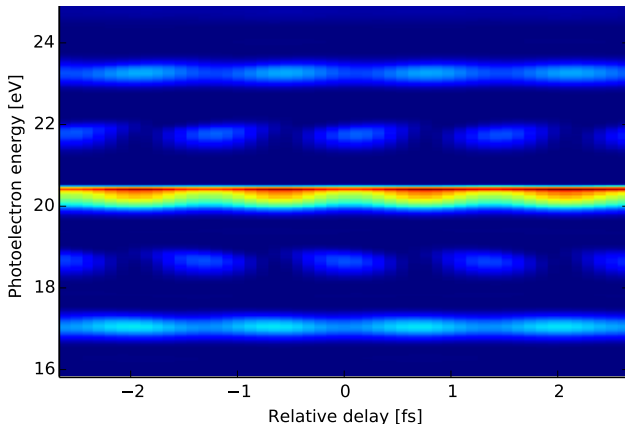
One photon absorption to dressed continuum with autoionization



Redistribution of three harmonic peaks due to laser dressing and autoionizing resonance ($q = -2.75$, $\Delta\epsilon = 0.2\omega_L$ and $\Gamma = 0.1\text{eV}$).

Photoelectron spectrogram

One photon absorption to dressed continuum with autoionization



Redistribution of three harmonic peaks due to laser dressing and autoionizing resonance ($q = -2.75$, $\Delta\epsilon = 0.2\omega_L$ and $\Gamma = 0.1\text{eV}$).

Conclusion and Outlook:

- **Attosecond pulse metrology** has shifted focus to make connection with the field of **theoretical atomic physics**.
- The simple approximations based on SFA are not sufficient to describe attosecond photoelectron dynamics.
- The **Wigner delay can not be directly measured**, but it can be extracted based on assumptions regarding the interaction with the probe field.
- Inter-species delay experiment show the best agreement with theory. More data on inter-orbital delays are needed.
- **Non-linear interaction with the fields and ion.**

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 - Anne L'Huillier
 - Johan Mauritsson
 - Kathrin Klünder
 - Diego Guénot + *et al.*
- **Stockholm University (SU)**
 - Eva Lindroth
 - Thomas Carette
- **Université Pierre et Marie Curie (UPMC)**
 - Alfred Maquet
 - Richard Taïeb

Thank you for your attention!